



Energy dependence of space-time extent of pion source in nuclear collisions

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May 1, 2015

Abstract

Energy dependence of space-time parameters of pion emission region at freeze-out is studied for collisions of various ions and for all experimentally available energies. The using of femtoscopic radii scaled on the averaged radius of colliding ions is suggested. This approach allows the expansion of the set of interaction types, in particular, on collisions of non-symmetrical ion beams which can be studied within the framework of common treatment. There is no sharp changing of femtoscopic parameter values with increasing of initial energy. Analytic functions suggested for smooth approximations of energy dependence of femtoscopic parameters demonstrate reasonable agreement with most of experimental data at $\sqrt{s_{NN}} \geq 5$ GeV. Estimations of some observables are obtained for energies of the LHC and FCC project.

1 Introduction

At present femtoscopic measurements in particular that based on Bose–Einstein correlations are unique experimental method for the determination of sizes and

lifetimes of sources in high energy and nuclear physics. The study of nucleus-nucleus (AA) collisions in wide energy domain by correlation femtoscopy seems important for better understanding both the equation of state (EOS) of strongly interacting matter and general dynamic features of soft processes. The discussion below is focused on specific case of femtoscopy, namely, on correlations in pairs of identical charged pions with small relative momenta – HBT-interferometry – in nucleus-nucleus collisions. The general view for phenomenological parameterization of correlation function (CF) for two identical particles is discussed in the [1, 2]. Below the experimental results obtained for AA collisions within the standard 3d approach are taken into account [2]. The set of main femtoscopic observables $\mathcal{G}_1 \equiv \{\mathcal{G}_1^i\}_{i=1}^4 = \{\lambda, R_s, R_o, R_l\}$ is under consideration as well as the set of some important additional observables which can be calculated with help of HBT radii $\mathcal{G}_2 \equiv \{\mathcal{G}_2^j\}_{j=1}^3 = \{R_o/R_s, \delta, V\}$. Here $\delta = R_o^2 - R_s^2$, $V = (2\pi)^{3/2} R_s^2 R_l$ is the volume of source at freeze-out. The set of parameters \mathcal{G}_1 characterizes the correlation strength and source's 4-dimensional geometry at freeze-out stage completely. The most central collisions are usually used for study the space-time characteristics of final-state matter, in particular, for discussion of global energy dependence of femtoscopic observables. Therefore scaled parameters \mathcal{G}_1^i , $i = 2 - 4$, δ and \mathcal{G}_2^3 are calculated as follows [1, 2]:

$$R_i^n = R_i/R_A, \quad i = s, o, l; \quad \delta^n = \delta/R_A^2; \quad V^n = V/V_A. \quad (1)$$

Here $R_A = R_0 A^{1/3}$, $V_A = 4\pi R_A^3/3$ is radius and volume of spherically-symmetric nucleus, $R_0 = (1.25 \pm 0.05)$ fm [3, 4]. The change $R_A \rightarrow \langle R_A \rangle = 0.5(R_{A_1} + R_{A_2})$ is made in the relation (1) in the case of non-symmetric nuclear collisions [1, 2]. In general case the scale factor in (1) should takes into account the centrality of nucleus-nucleus collisions. The normalization procedure suggested in [1] allows the consideration of all available data for nucleus-nucleus collisions [2]. As development of previous analyses [1, 2] the proton-proton (pp) results at high energies [5, 6] are also considered here with replacing $R_A \rightarrow R_p$ in (1).

2 Energy dependence of space-time extent of pion source

Detail study for (quasi)symmetric heavy ion collisions [1, 2] demonstrates that the fit function ($\varepsilon \equiv s_{NN}/s_0$, $s_0 = 1$ GeV²)

$$f(\sqrt{s_{NN}}) = a_1 [1 + a_2 (\ln \varepsilon)^{a_3}] \quad (2)$$

agrees reasonably with experimental dependence $\mathcal{G}_1^i(\sqrt{s_{NN}})$, $i = 1 - 4$ at any collision energy for λ and at $\sqrt{s_{NN}} \geq 5$ GeV for HBT radii. Fig. 1 shows the energy dependence of λ (a), scaled HBT-radii (b – d) and R_o/R_s ratio (e) for both the symmetric and non-symmetric collisions of various nuclei. Fits of experimental dependencies for AA interactions are made by (2) in the same energy domains as well as for (quasi)symmetric heavy ion collisions. The numerical values of fit parameters are presented in Table 1, fit curves are shown in Fig. 1 by solid lines

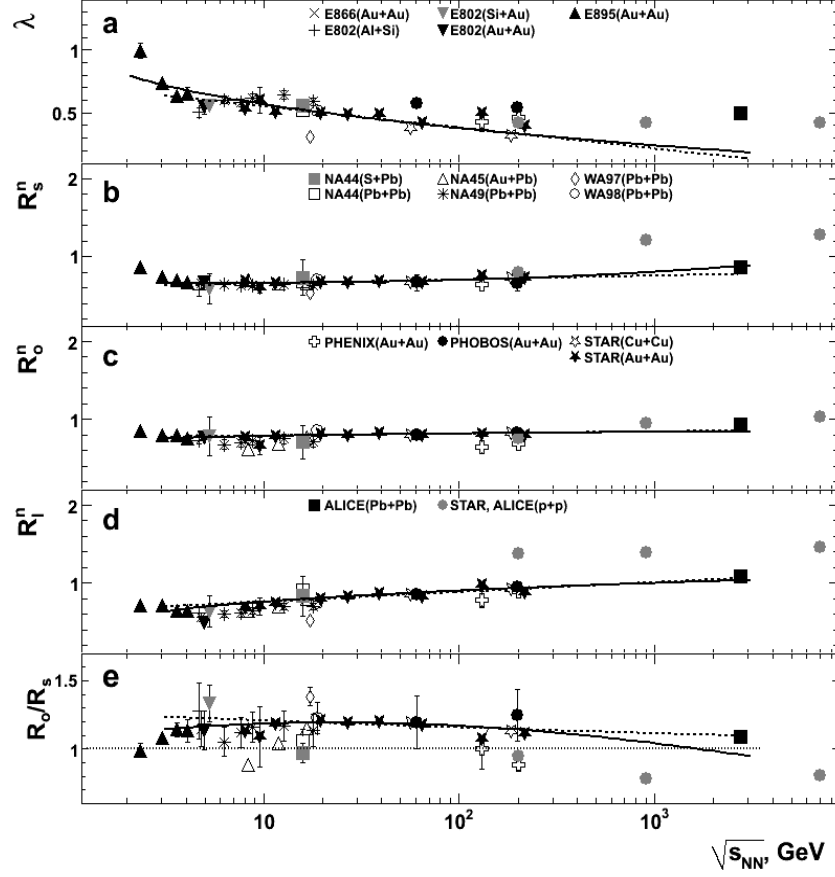


Figure 1: Energy dependence of λ parameter (a), scaled HBT-radii (b – d) and ratio R_o/R_s (e) in various collisions. Experimental data are from [2, 5, 6]. Statistical errors are shown (for NA44 – total uncertainties). The solid lines (a – d) correspond to the fits by function (2) and dashed lines – to the fits by specific case of (2) at fixed $a_3 = 1.0$. Smooth solid and dashed curves at (e) correspond to the ratio R_o/R_s calculated from the fit results for R_s^n and R_o^n in AA, dotted line is the level $R_o/R_s = 1$.

for (2) and by dashed lines for specific case of fit function at $a_3 = 1.0$ with taking into account statistical errors. There is dramatic growth of $\chi^2/\text{n.d.f.}$ values for fits of λ data (Fig. 1a) despite of qualitative agreement between smooth approximations and experimental λ values for range $10 \lesssim \sqrt{s_{NN}} \lesssim 200$ GeV. The fit by (2) underestimates the λ value at the LHC energy $\sqrt{s_{NN}} = 2.76$ TeV significantly. The λ values for asymmetric nucleus-nucleus collisions at intermediate energies $\sqrt{s_{NN}} \lesssim 20$ GeV agree well with values of λ in symmetric heavy ion collisions at close energies. On the other hand the λ for Cu+Cu collisions is smaller

Table 1: Values of fit parameters for AA data with statistical errors

HBT parameter	Fit parameter			
	a_1	a_2	a_3	$\chi^2/\text{n.d.f.}$
λ	1.21 ± 0.09	-0.30 ± 0.04	0.38 ± 0.04	3656/29
	0.717 ± 0.003	-0.051 ± 0.001	1.0 (fixed)	3786/23
R_s^n	0.656 ± 0.002	$(6 \pm 3) \times 10^{-5}$	3.11 ± 0.19	195/25
	0.599 ± 0.003	0.019 ± 0.001	1.0 (fixed)	280/26
R_o^n	0.10 ± 0.02	6.3 ± 1.7	0.068 ± 0.006	402/25
	0.758 ± 0.004	0.008 ± 0.001	1.0 (fixed)	415/26
R_l^n	0.022 ± 0.002	23 ± 3	0.258 ± 0.005	502/25
	0.634 ± 0.004	0.043 ± 0.001	1.0 (fixed)	615/26

systematically than λ in Au+Au collisions in energy range $\sqrt{s_{NN}} = 62 - 200$ GeV (Fig. 1a). New experimental data are important for verification of the suggestion of separate dependencies $\lambda(\sqrt{s_{NN}})$ for moderate and heavy ion collisions. Also the development of some approach is required in order to account for type of colliding beams in the case of λ parameter and improve quality of approximation. Smooth curves for normalized HBT radii and ratio R_o/R_s are in reasonable agreement with experimental dependencies in fitted domain of collision energies $\sqrt{s_{NN}} \geq 5$ GeV (Figs. 1b – e). Dramatic improvement of the fit qualities for scaled HBT radii at transition from the data sample with statistical errors to the data sample with total errors is dominated mostly by the uncertainty in r_0 leads to additional errors due to scaling (1). The scaled HBT-radii in pp are larger significantly than those in AA collisions at close energies. Because feature of Regge theory [7] the following relation is suggested to take into account the expanding of proton with energy: $R_p = r_0(1 + k\sqrt{\alpha'_p \ln \varepsilon})$, where $r_0 = (0.877 \pm 0.005)$ fm is the proton's charge radius [8], parameter $\alpha'_p \propto \ln \varepsilon$ because of diffraction cone shrinkage speeds up with collision energy in elastic pp scattering [9]. The k is defined from the boundary condition $R_p \rightarrow 1/m_\pi$ at $\varepsilon \rightarrow \infty$ with choice of appropriate asymptotic energy $\sqrt{s_{NN}^a}$. The detail study demonstrates that the increasing of $\sqrt{s_{NN}^a}$ from 6 PeV [10] to 10^3 PeV influences weakly on R_i^n , $i = s, o, l$ in pp collisions and calculations are made for the first case. The normalized transverse radii agree in both the pp and the AA collisions (Figs. 1b, c) at $\sqrt{s_{NN}} = 200$ GeV with excess of R_s^n in pp with respect to the AA in TeV-region. The R_l^n in pp is larger than that for AA in domain $\sqrt{s_{NN}} \geq 200$ GeV. It should be stressed that the additional study is important, at least, for choice of $R_p(\varepsilon)$.

The corresponding dependencies for δ^n and V^n are demonstrated in Fig. 2 and Fig. 3, respectively. As well as in [1, 2] results for $\pi^+\pi^+$ pairs are shown in Figs. 1 – 3 also because femtoscopy parameters from the set \mathcal{G}_1 depend on sign of electrical charge of secondary pions weakly. The relation $R_o < R_s$ is observed for $\approx 11\%$ of points in Fig. 2. Detail discussion for points with $\delta < 0$ is in the [2]. The

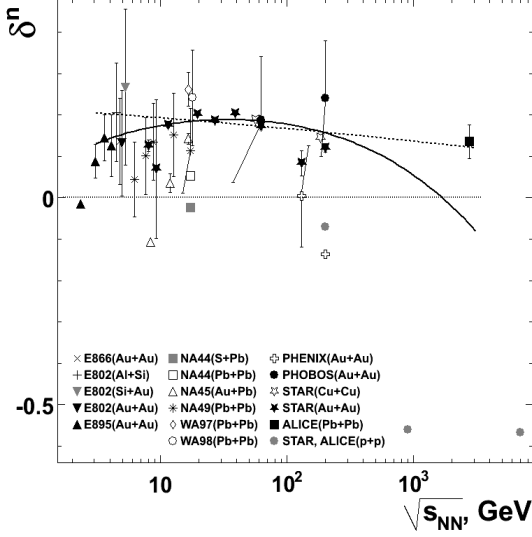


Figure 2: Dependence $\delta^n(\sqrt{s_{NN}})$ in various collisions. Experimental data are from [2, 5, 6]. Error bars are only statistical (for NA44 – total uncertainties). Dotted line is the level $\delta^n = 0$. Smooth curves are derived from (1) and the fit results for R_s^n , R_o^n in AA. The solid line corresponds to the fits of normalized HBT radii by function (2) and dashed line – to the fits by specific case $R_i^n \propto \ln \varepsilon$, $i = s, o$.

dependence $\delta^n(\sqrt{s_{NN}})$ is almost flat within large error bars in all energy domain under consideration. Taking into account the STAR high-statistics results [11] only one can see the indication on change of behavior of $\delta^n(\sqrt{s_{NN}})$ inside the range of collision energy $\sqrt{s_{NN}} = 11.5 - 19.6$ GeV. This observation is in agreement with features of behavior of emission duration ($\Delta\tau$) dependence on $\sqrt{s_{NN}}$ discussed in [2]. The estimation of energy range agrees well with results of several studies in the framework of the phase-I of the beam energy scan (BES) program at RHIC which indicate on the transition from dominance of quark-gluon degrees of freedom to hadronic matter at $\sqrt{s_{NN}} \lesssim 19.6$ GeV. But future precise measurements are crucially important for extraction of more definite physics conclusions. Smooth solid and dashed curves shown in Fig. 2 are calculated for δ^n from the fit results for R_s^n and R_o^n (Table 1). The calculation based on the fit function (2) at free a_3 agrees reasonably with experimental points at $\sqrt{s_{NN}} \leq 200$ GeV but underestimates δ^n in TeV-region significantly. The large errors in Fig. 3 for strongly asymmetric AA collisions is dominated by large difference of radii of colliding moderate and heavy nuclei and corresponding large uncertainty for $\langle R_A \rangle$. Smooth solid and dashed curves shown in Fig. 3 are calculated for V^n from it's definition (1) and the fit results for R_s^n , R_l^n (Table 1). The fit results for normalized HBT radii obtained with general function (2) lead to very good agreement between smooth curve and experimental

data in TeV-region in contrast with the curve obtained from corresponding fit results for (2) at $a_3 = 1.0$. There is significant difference between pp and AA collisions for δ^n in TeV-region (Fig. 2) and for V^n at $\sqrt{s_{NN}} \geq 200$ GeV (Fig. 3).

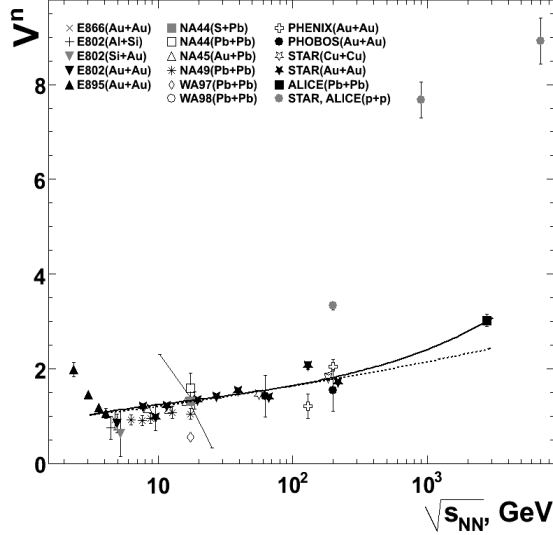


Figure 3: Energy dependence of V^n in various collisions. Experimental data are from [2, 5, 6]. Error bars are only statistical (for NA44 – total uncertainties). Smooth curves are derived from (1) and the fit results for R_s^n , R_l^n in AA. The solid line corresponds to the fits of normalized HBT radii by function (2) and dashed line – to the fits by specific case $R_i^n \propto \ln \varepsilon$, $i = s, l$.

Estimations for λ , R_o/R_s , and normalized femtoscopic parameters at the LHC and the FCC energies are shown in Table 2 for fits of various AA collisions with inclusion of statistical errors, the second line for each collision energy corresponds to the using of the specific case of (2) at $a_3 = 1.0$. All the smooth approximations discussed above predict amplification of coherent pion emission with significant decreasing of λ . Uncertainties are large for estimations obtained on the basis of results of fits by function (2) at free a_3 . Thus values of femtoscopic observables in Table 2 are equal within errors for general and specific case of (2) at $\sqrt{s_{NN}} = 5.52$ TeV (LHC) and $\sqrt{s_{NN}} = 39.0$ TeV (FCC).

The energy dependencies for sets \mathcal{G}_m , $m = 1, 2$ of femtoscopic parameters with taking into account the scaling relation (1) demonstrate the reasonable agreement between values of parameters obtained for interactions of various ions (Figs. 1 – 3). The observation confirms the suggestion [1] that normalized femtoscopic parameters allow us to unite the study both the symmetric and the asymmetric AA collisions within the framework of united approach. This qualitative suggestion is confirmed indirectly by recent study of two-pion correlations in the collisions of the lightest nucleus (d) with heavy ion (Au) at RHIC. Estimations of space-time extent of the

Table 2: Estimations for observables based on fit results

$\sqrt{s_{NN}}$, TeV	HBT parameter for AA			
	λ	R_s^n	R_o^n	R_l^n
5.52	0.16 ± 0.19	0.9 ± 0.2	0.8 ± 0.3	1.06 ± 0.16
	0.091 ± 0.004	0.792 ± 0.009	0.860 ± 0.010	1.099 ± 0.013
39.0	0.07 ± 0.21	1.2 ± 0.4	0.9 ± 0.3	1.11 ± 0.16
	–	0.836 ± 0.011	0.883 ± 0.012	1.205 ± 0.015
	R_o/R_s	δ^n	V^n	
5.52	0.9 ± 0.4	-0.2 ± 0.6	3.5 ± 1.6	
	1.086 ± 0.018	0.11 ± 0.02	2.59 ± 0.07	
39.0	0.7 ± 0.3	-0.7 ± 1.1	6 ± 4	
	1.06 ± 0.02	0.08 ± 0.03	3.17 ± 0.09	

pion emission source in d+Au collisions at top RHIC energy [12] in dependence on kinematic observables show similar patterns with corresponding dependencies in Au+Au collisions and indicate on similarity in expansion dynamics in collisions of various systems (d+Au and Au+Au at RHIC, p+Pb and Pb+Pb at LHC). The scaling results for some radii indicate that hydrodynamic-like collective expansion is driven by final-state rescattering effects [12]. On the other hand the normalized femtoscopic parameters allow us to get the common kinematic dependencies only without any additional information about possible general dynamic features in different collisions. Thus the hypothesis discussed above is qualitative only. The future quantitative theoretical and phenomenological studies are essential for verification of general features of soft stage dynamics for different collisions at high energies.

3 Summary

The main results of present study are the following.

Energy dependence is investigated for range of all experimentally available initial energies and for estimations of the main femtoscopic parameters from set the \mathcal{G}_1 (λ and radii) derived in the framework of Gauss approach as well as for the set of important additional observables \mathcal{G}_2 contains ratio of transverse radii, δ and HBT volume. There is no dramatic change of femtoscopic parameter values in AA with increasing of $\sqrt{s_{NN}}$ in domain of collision energies $\sqrt{s_{NN}} \geq 5$ GeV. The energy dependence is almost flat for the δ^n in nucleus-nucleus collisions within large error bars. The indication on possible curve knee at $\sqrt{s_{NN}} \sim 10 - 20$ GeV obtained in the STAR high-statistics data agree with other results in the framework of the phase-I of the BES program at RHIC. But additional precise measurements are crucially important at various $\sqrt{s_{NN}}$ in order to confirm this feature in energy dependence of additional femtoscopic parameters. The normalized some HBT radii

and source volume in pp are larger significantly than those in AA collisions especially in TeV-region. The fit curves demonstrate qualitative agreement with experimental AA data for λ at all available collision energies and for normalized HBT radii in energy domain $\sqrt{s_{NN}} \geq 5$ GeV. Smooth curves calculated for energy dependence of parameters from the set \mathcal{G}_2 agree reasonably with corresponding experimental AA data in the most cases. Estimations of femtoscopic observables are obtained on the basis of the fit results for energies of the LHC and the FCC project. For multi-TeV energy domain the emission region of pions in nucleus-nucleus collisions will be characterized by decreased correlation strength, linear sizes about $8.5 - 9.5$ fm in longitudinal direction and $7 - 8$ fm in transverse plane, volume of about 10^4 fm³.

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